



MATHEMATICS TEACHER'S MAGAZINE: CONTRIBUTIONS OF ANGLES THEME FOR THE GEOMETRY TEACHING

REVISTA DO PROFESSOR DE MATEMÁTICA: CONTRIBUIÇÕES DO TEMA 'ÂNGULOS' PARA O ENSINO DE GEOMETRIA

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Abstract

Geometry is an important field of Mathematics and its teaching can be favored through the contributions of mathematics magazines to didactic work in the classroom. The objective of this text was mapping the contributions of papers published regarding the theme 'Angles' in Geometry teaching at Middle School. The methodology was situated at the documentary modality and the research corpus was the Mathematics Teacher's Magazine. We observed that the most developed Geometry contents on the Mathematics Teacher's Magazine texts were triangles, solids, polygons, geometrical constructions and circle. Regarding to texts themes, we realized that the main discourses were around the curriculum, epistemology, didactics and teacher education in the segment of the Middle School. The theme angles had a thiny percentage over papers that talked about Geometry in the Mathematics Teacher's Magazine. Concerning the texts that permeated the subtheme 'Angles' 22.22% dealt with 'Angles in the Circumference', 16.67% placed 'Direct Proportionality', 16.67% the 'Sexagesimal System', 11.10% dealt with 'Parallelism between strk aights', 11.10% the 'Tales Theorem', 5.56%' the 'Sum of Angles in a Triangle', 5.56% the 'Pythagoras Theorem' and 5.56% to 'Trigonometry in Right triangle'.

Keywords: Angle. Geometry. Teaching. Mathematics Teacher's Magazine.

Resumo

A Geometria é um importante campo da Matemática e seu ensino pode ser favorecido por meio das contribuições de revistas de matemática para o trabalho didático em sala de aula. O objetivo deste texto foi inventariar contribuições de artigos publicados no que concerne ao tema 'Ângulos', no âmbito do ensino de Geometria nos anos finais do Ensino Fundamental. A metodologia se situou na modalidade documental e o *corpus* foi a Revista do Professor de Matemática. Observamos que os conteúdos mais desenvolvidos foram triângulos, sólidos, polígonos, construções geométricas e círculo. Quanto à temática dos artigos, percebemos que os principais discursos se situavam em torno do currículo, da epistemologia, da didática e da formação de professor no segmento dos anos finais do Ensino Fundamental brasileiros. O tema dos ângulos teve uma porcentagem ínfima sobre os trabalhos que falavam sobre Geometria na Revista do Professor de Matemática. Quanto aos artigos que permeavam o subtema 'ângulos' 22,22% tratavam de 'Ângulos na Circunferência', 16,67% situavam a 'Proporcionalidade direta', 16,67% o 'Sistema sexagesimal', 11,10% tratavam do 'Paralelismo entre retas', 11,10% o 'Teorema de Tales', 5,56% a 'Soma dos ângulos em um triângulo', 5,56% o 'Teorema de Pitágoras' e 5,56% a 'Trigonometria no Triângulo Retângulo'.

Palavras-chave: Ângulos. Ensino. Geometria. Revista do Professor de Matemática

1 INTRODUÇÃO

Boyer (1995) tells us that aspects of Geometry can be found in some stone inscriptions and other artifacts since the beginning of man's appearance on Earth Planet. For the author, in great civilizations, like ancient Babylonians and Egyptians, Geometry acquired characteristics linked to the pragmatic world.

The etymology of the term Geometry comes from the Greek meaning 'to measure the land', which refers to pragmatic activities. The first measurements used were those relative to the human body (feet, hands, among others parts of human bodies). The first rulers were made of wood, metal or ropes subdivided by knots. Constructions generally involved right angles, but there was an evolution of non-rectangular shapes, which required some angles knowledge.

According to Boyer (1991), it was in Ancient Greece that Geometry acquired an abstract character, reaching a peak in Euclid's Elements, a set of thirteen chapters, ten of which are dedicated to Geometry. This work was considered one of the most important in terms of Geometry knowledge of ancient civilizations.

D'Ambrosio (1993) considers that the rigor initiated by Euclid's work represented an essential moment in the study of Mathematics, which remained strongly inserted in teaching until the so-called Modern Mathematics Movement (which emerged since mid-50's to 70's), which was concerned with teaching Mathematics with rigor and formality.

After some criticism regarding the Modern Mathematics Movement for its inefficiency and insufficiency in the field of didactics, Silva, Gomes and Silva (2021) point out that Geometry teaching has called for changes, so as not to prioritize rigor for the basic education.

Considering Geometry teaching at basic education, some teachers have raised concerns about the appropriate material for this subject. In this sense, the "[...] search for new forms and pedagogical practices to restore quality in Geometry teaching has been highlighted in the work of researchers around the entire world"¹ (Bayer; Lobo, 2005, p. 15).

Nowadays, Geometry is a significant language for developing scientific thinking in a technological and globalized world that increasingly demands much more knowledge in all areas. In this sense, there is a concern to rescue the Geometry teaching:

[...] as one of the fundamental areas of Mathematics, which has led many teachers and researchers to dedicate themselves to reflection and the elaboration, implementation and evaluation of alternatives, which seek to overcome the difficulties often encountered in approaching this theme, in basic school or at higher levels of education² (Fonseca *et al.*, 2001, p. 91).

We hypothesize that Mathematics magazines represent an item to be investigated, especially in terms of being an important material that is available on the internet and can contribute to the preparation of Mathematics classes at basic education.

¹ “[...] busca de novas formas e práticas pedagógicas para se resgatar o ensino de Geometria com qualidade tem sido destaque em trabalhos de pesquisadores em todo o mundo”.

² “[...] resgatar o ensino da geometria como uma das áreas fundamentais da matemática, o que tem levado muitos professores e pesquisadores a se dedicarem à reflexão e à elaboração, implementação e avaliação de alternativas, que busquem superar as dificuldades não raro encontradas na abordagem desse tema, na escola básica ou em níveis superiores de ensino (FONSECA *et al.*, 2001, p. 91).

The Mathematics Teacher's Magazine³ is a publication that provides papers and reference material at an elementary or advanced level, which is accessible to Elementary School, Middle School and High School Mathematics teachers and yet to students of Bachelor's Degree in Mathematics and related courses.

So, this paper's objective was mapping the contributions of papers published regarding the theme 'Angles' in Geometry teaching at Middle School.

2 BRAZILIAN CURRICULAR DOCUMENTS AND TEACHING ANGLES

Some official Brazilian documents, such as the National Curricular Parameters, described in Brazil (1997), highlight that the study of Geometry is a fertile field for working with various teaching resources, such as contextualization and problematization. The aforementioned document states that working with notions of Geometry can collaborate with the teaching and learning of aspects involving numbers and measurements, as it encourages the act of observing, noticing similarities and differences, identifying regularities, among other essential educational aspects associated with basic education.

The National Curricular Parameters placed new value on the field of teaching, since “[...] Mathematics should be seen by the student as a knowledge that can favor the development of his reasoning, his expressive capacity, his aesthetic sensitivity and his imagination”⁴ (Brazil, 1997, p.26).

The aforementioned document treated Geometry with great emphasis, placing it on two axes: ‘Space and Form’ and ‘Quantities and Measures’.

Geometric concepts constitute an important part of the Mathematics curriculum in Elementary and Middle School, because, through them, the student develops a special type of thinking that allows them to understand, describe and represent, in an organized way, the world in which they live⁵ (Brazil, 1997, p. 39).

Almost two decades later, the National Common Curricular Base, described in Brazil (2018), the most current curricular document in the Education area, highlights that the Geometry is linked to the fields of Arithmetic, Algebra, Statistics and Probability.

For this document, the teaching of Geometry “[...] involves the study of a broad set of concepts and procedures necessary to solve problems in the physical world and different areas of knowledge”⁶ (Brazil, 2018, p. 269).

³ In Portuguese, ‘Revista do professor de Matemática’ (RPM), located at São Paulo, Brazil, South America.

⁴ “[...] Matemática deverá ser vista pelo aluno como um conhecimento que pode favorecer o desenvolvimento do seu raciocínio, de sua capacidade expressiva, de sua sensibilidade estética e de sua imaginação”.

⁵ Os conceitos geométricos constituem parte importante do currículo de Matemática no Ensino Fundamental, porque, por meio deles, o aluno desenvolve um tipo especial de pensamento que lhe permite compreender, descrever e representar, de forma organizada, o mundo em que vive.

⁶ “[...] envolve o estudo de um amplo conjunto de conceitos e procedimentos necessários para resolver problemas do mundo físico e de diferentes áreas do conhecimento”.

Inherent to the branch of Geometry, it allows students to develop their geometric thinking. This type of thinking:

[...] is necessary to investigate properties, to make conjectures, and to produce convincing geometric arguments. It is also important to consider the functional aspect that must be present in the study of Geometry: geometric transformations, especially symmetries. The fundamental mathematical ideas associated with this theme are, mainly, construction, representation and interdependence⁷ (Brazil, 2018, p. 269).

Shirlo and Da Silva (2013) point out that in different Mathematics curricula, the notion of angle “[...] constitutes a key concept for the study of similar figures, cases of congruence of triangles, construction of regular polygons, metric relationships in a triangle, trigonometry, analytical geometry and spacial geometry”⁸ (p.190).

In particular, the National Common Curricular Base, described in Brazil (2018), with regard to the development of competences, proposed that the theme 'Angles' might be developed in the skills EF04MA18, EF05MA17, EF05MA18, EF06MA17, EF07MA19 and EF07MA22, as highlighted in Table 01.

Table 01: Skills correlated to the theme ‘Angles’.

Knowledge Objet	Skill
MATHEMATICS – FOURTH DEGREE	
“Right and non-right angles: use of folding, squares and software” (Brazil, 2018, p. 290).	“(EF04MA18) Recognize right and non-right angles in polygonal figures using folding, squares or geometry software” (Brazil, 2018, p. 291).
MATHEMATICS – FIFTH DEGREE	
“Flat geometric figures: characteristics, representations and angles” (Brazil, 2018, p. 294).	“(EF05MA17) Recognize, name and compare polygons, considering sides, vertices and angles , and draw them, using drawing material or digital technologies” (Brazil, 2018, p. 295).
“Enlarging and reducing polygonal figures in gridded geometric meshes: recognition of the congruence of angles and proportionality from the corresponding sides” (Brazil, 2018, p. 294).	“(EF05MA18) Recognize the congruence of angles and the proportionality between the corresponding sides of polygonal figures in situations of enlargement and reduction in gridded geometric meshes and using digital technologies” (Brazil, 2018, p. 295).
MATHEMATICS – SIXTH DEGREE	
“Polygons: classifications regarding the number of vertices, the measurements of sides and angles and the parallelism and perpendicularity of the sides” (Brazil, 2018, p. 300).	“(EF06MA18) Recognize, name and compare polygons, considering sides, vertices and angles , and classify them as regular and non-regular, both in their representations in the plane and in faces of polyhedra. (EF06MA19) Identify characteristics of triangles and classify them according to the measurements of sides and angles . (EF06MA20) Identify characteristics of quadrilaterals, classify them in relation to sides and angles and recognize the inclusion and intersection of classes between them”

⁷ [...] é necessário para investigar propriedades, fazer conjecturas e produzir argumentos geométricos convincentes. É importante, também, considerar o aspecto funcional que deve estar presente no estudo da Geometria: as transformações geométricas, sobretudo as simetrias. As ideias matemáticas fundamentais associadas a essa temática são, principalmente, construção, representação e interdependência.

⁸ “[...] se constitui num conceito chave para o estudo de figuras semelhantes, casos de congruência de triângulos, construção de polígonos regulares, relações métricas num triângulo, trigonometria, geometria analítica e geometria espacial”.

	(Brazil, 2018, p. 301).
MATHEMATICS – SEVENTH DEGREE	
“Relationships between angles formed by parallel lines intersected by a transversal” (Brazil, 2018, p. 306).	“(EF07MA23) Check relationships between angles formed by parallel lines cut by a transversal, with and without the use of dynamic geometry software” (Brazil, 2018, p. 307).
“Regular polygons: square and equilateral triangle” (Brazil, 2018, p. 306).	“(EF07MA27) Calculate measurements of internal angles of regular polygons, without using formulas, and establish relationships between internal and external angles of polygons, if possible linked to the construction of mosaics and tiling, the manufacture of tools and mechanical parts, among others” (Brazil, 2018, p. 307).
MATHEMATICS – EIGHT DEGREE	
“Geometric constructions: angles of 90°, 60°, 45° and 30° and regular polygons” (Brazil, 2018, p. 312).	“(EF08MA15) Build, using drawing instruments or dynamic geometry software, perpendicular bisectors, bisectors, angles of 90°, 60°, 45° and 30° and regular polygons. (EF08MA16) Describe, in writing and through a flowchart, an algorithm for constructing a regular hexagon of any area, based on the measurement of the central angle and the use of squares and compasses” (Brazil, 2018, p. 313).
MATHEMATICS – NINTH DEGREE	
“Demonstrations of relationships between angles formed by parallel lines intersected by a transversal” (Brazil, 2018, p. 314).	“(EF09MA10) Demonstrate simple relationships between the angles formed by parallel lines cut by a transversal” (Brazil, 2018, p. 315).
“Relationships between arcs and angles on the circumference of a circle” (Brazil, 2018, p. 314).	“(EF09MA11) Solve problems by establishing relationships between arcs, central angles and angles inscribed on the circumference, including using dynamic geometry software” (Brazil, 2018, p. 315).

Source: The author, based on Brazil (2018).

Based on Table 01 we observed that the theme 'Angles' permeates several moments of Elementary and Middle School, which is an indicative factor of the importance derivated from this topic. This characterizes the spiral curriculum, inspired by Jerome Bruner's ideas. For Bruner (1987), the opportunity to revisit themes at different times becomes important, as there is greater interaction between subjects, as well as rescuing and expanding gradually the acquisition and development of knowledge.

Yet, Table 01 places the theme of 'Angles' in a spiral curriculum motto, as relates this subject to polygonal figures, polygons (triangles, quadrilaterals, among others), relative positions between straight lines, the use of construction processes with a protractor, ruler, square and compasses, as well as in the exploration of dynamic software, places where the intramathematical connection can be observed.

In terms of applicability, the aforementioned Brazilian curricular document places connections with tools and mechanical devices, as well as in the study of tiling flat surfaces. Next, we highlight the characteristics of the methodology used in this research.

3 METHODOLOGY

The methodology used in this research was documentary, within the quali-quantitative perspective. Flick (2009) highlights that in several areas there is a growth in research that emphasizes possible relationships or combinations between quantitative and qualitative research. For the aforementioned author, qualitative-quantitative research is based on the description, classification and interpretation of empirical information and the analysis of statistics and numerical data, in order to enable a more general framework involving certain study.

Grácio and Garrutti (2005) argue that in the Education area, the quali-quantitative articulation enables a broader conception of the phenomena researched, so that “[...] quantifications strengthen arguments and constitute important indicators for qualitative analyses” (Grácio; Garrutti, 2005, p. 119).

Gatti (2004) considers that the quali-quantitative approach tends to demystify representations and preconceptions in educational phenomena, so that numerical data “[...] can be very useful in understanding various educational problems. Furthermore, the combination of this type of data with data from qualitative methodologies can enrich the understanding of events, facts and processes” (p.4).

Sá-Silva, Almeida and Guindani (2009) highlight that the use of documents in research must be appreciated and valued. So, documentary research seeks to “[...] identify factual information in documents based on questions and hypotheses of interest”⁹ (Sá-Silva; Almeida; Guindani, 2009, p. 3).

Cellard (2008) and Sá-Silva; Almeida; Guindani (2009) highlight that the positivist school had adopted the written record as an official means of research at the end of the 19th century, as the document was seen as a type of proof, a guarantee of fidelity and, furthermore, there was the aspect of objectivity associated with this type of material.

According to Appolinário (2009), documentary research is based on sources from written or unwritten texts, such as books, magazines, electronic media files, films, videos, among other ways. Gil (2007) added that documentary research may use quantitative or qualitative data, which depends on the research’s objective. For Figueiredo (2007), documents can be used as sources of information, indications and clarifications that, in their essence, can elucidate certain questions, prove other conjectures or clarify some point that is in the search or interest of a certain researcher.

This research situated a documentary investigation, considering the theoretical contributions of several authors who are specialized in Mathematics teaching, who observe updates in the strategies used to work on Geometry in Elementary and Middle School.

The material researched in this paper was circumscribed to the Mathematics Teacher’s Magazine, published by the Brazilian Mathematics Society since 1982, and which proposes some mathematical texts and papers aimed at the classroom work on different themes and approaches, according to Moriel Junior and Wielewski (2013) and Santos (2019).

According to the Mathematics Teacher’s Magazine’, available at https://rpm.org.br/default.aspx?m_id=4, the intention of the publication is to serve

⁹ “[...] identificar informações factuais nos documentos a partir de questões e hipóteses de interesse”.

professionals who teaches Mathematics, especially those who works with the final grades of Elementary School, Middle School and High School. This magazine publishes elementary and advanced texts, but with the intention of being accessible to school teachers or students of Mathematics degree courses.

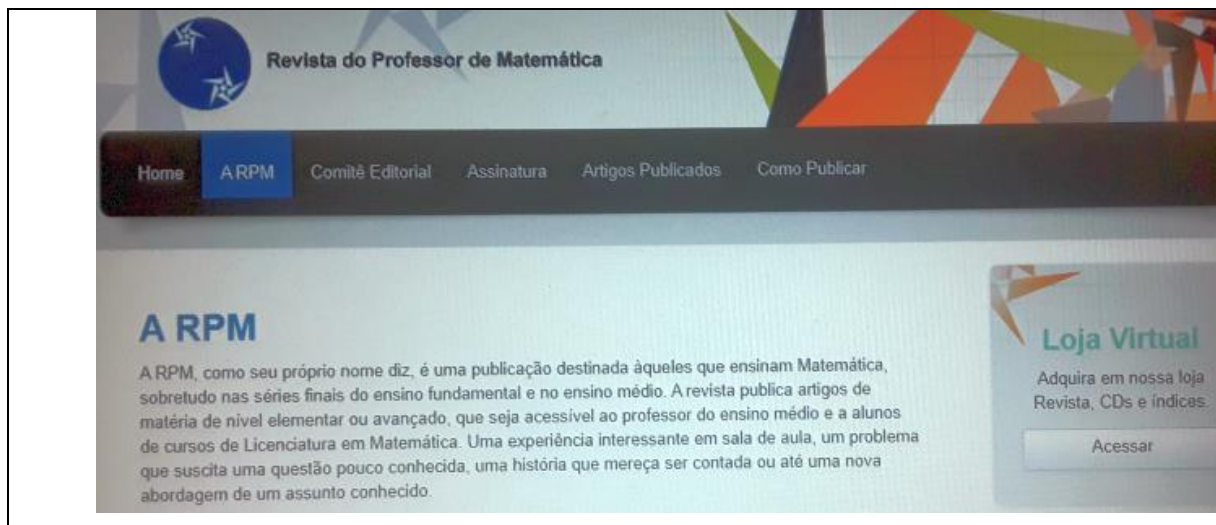
Moriel Junior and Wielewski (2013) highlight some aspects present at the publications of the Mathematics Teacher's Magazine, because its importance for the teachers. So its mentioned that texts of this magazine directly [...] address topics at an elementary or advanced level, experiences in the classroom, problems that raise little-known questions, new approaches of known subjects, among other topics (as described on the journal's website) (Moriel Junior; Wielewski, 2013, p. 977-978).

Also, Santos (2019) points out that since the time of its first number (in 1982), the majority community of Mathematics teachers was in need of a periodical focused on some epistemological and didactical contributions to Basic Education. In this way, the emergence of the Mathematics Teacher's Magazine represented the proposition “[...] of a vehicle in which those who teach the subject of Mathematics can suppress their doubts and gain more knowledge about the subject”¹⁰ (Santos, 2019, p. 169).

The Mathematics Teacher's Magazine was proposed at meetings of the Brazilian Mathematics Society, organization that aimed to discuss teaching problems in that subject. According to Santos (2019), the idea of creating this periodical arose from meetings between mathematicians and mathematics teachers, whose purpose was to establish a democratic communication vehicle among Brazilian Mathematics Society and the Mathematics teachers.

In this scenario, the magazine's motto was to provide an interesting experience in the classroom, a problem that could revive a little-known issue, a story that deserved to be (re)told or even a new approach to certain mathematical subjects. In Table 02 we present the platform and the Mathematics Teacher's Magazine' design on the digital platform.

¹⁰ “[...] de um veículo no qual àqueles que ministram a disciplina de Matemática podem suprimir suas dúvidas e ter mais conhecimento sobre a disciplina”.

Table 02: Design of the ‘Mathematics Teacher's Magazine’.

Source: Mathematics Teacher's Magazine (s.d.).

According to the website https://rpm.org.br/default.aspx?m_id=4, the Mathematics Teacher's Magazine seeks to present some texts and papers that are accessible and enjoyable to read, being a vehicle for the circulation and exchange of ideas and an instrument of stimulation to intellectual curiosity.

We turn to a possible characterization of the Mathematics Teacher's Magazine. For this endeavor we will use Peters' classification: reflection and consultation magazines. These material classification are related to papers which aimed to the teacher's continuing education.

Following the first conception, Peters (2005) considers that reflection magazines can enrich “[...] the theoretical framework and can be used by teachers who wish to follow the paths of the History of Mathematics in their classes” (p. 5). In this sense, we can extend this consideration to magazines that propose general theoretical references that can contribute to a repertoire that supports and justifies choices that the teacher can bring to classes preparation.

In turn, Peters (2005) considers that 'consultation magazines' offer “[...] the teacher material for direct use in the classroom”¹¹ (p. 5), in the sense that the text offers the teacher a work specific and directed to its immediate use in classroom, without necessarily offering a theoretical framework that justifies such procedures adopted by the papers' authors.

From our experience as a Mathematics Teacher's Magazine reader since 1982, we believe that this vehicle is, in essence, within the perspective of 'consultation magazines'. At times, there are texts that outline moments of reflection. For this reflection, we consider that the Mathematics Teacher's Magazine can represent a reference guide to support the teacher with materials for direct use in the classroom, since several texts and papers present reports of classes in which the results were obtained through the bias of Mathematics applicability, as well as suggested problems.

We believe that, sometimes, reading the Mathematics Teacher's Magazine can provide teachers with a movement to search for support material to prepare the class, in a different way

¹¹ “[...] ao professor material de utilização direta em sala de aula”.

than is presented by Brazilian textbooks. Thus, the papers presented in the aforementioned magazine can enrich the repertoire of material to be worked on by Mathematics teachers in the basic education classroom.

4 DATA COLLECTION, DESCRIPTION AND ANALYSIS STAGE

To data collection we accessed the Mathematics Teacher's Magazine website, available at <http://rpm.org.br/> and, more specifically, at '<http://rpm.org.br/BuscaAvancada.aspx>'. On April 3, 2021, the search keyword 'Geometry' was entered, plus the filter 'Group by subject'. Based on this initial mapping, we summarized the data in Table 03.

Table 03: The Geometry areas that was covered until edition 89.

Subject	Sub-theme	Quantity	Percentage
Geometry	Astronomy	6	1,76%
	Angles	9	2,64%
	Otthers	22	6,44%
	Teaching	23	6,74%
	History	31	9,06%
	Circles	31	9,06%
	Conics	31	9,06%
	Geometric Constructions	37	10,81%
	Polygons	44	12,86%
	Solids	52	15,20%
	Triangles	56	16,37%
	TOTAL	342	100%

Source: The author.

In this table we indicate the results in an increasing order of papers available on the Mathematics Teacher's Magazine Platform, through the classification of subtopics that the magazine itself had already categorized. From the data described in Table 03, we observed a wide range of Geometry subtopics. There is a relatively balanced distribution between most of the topics covered. The topics most covered by the Mathematics Teacher's Magazine were 'Polygons', 'Solids' and 'Triangles'.

Furthermore, Table 03 indicates that the theme 'angles' represents a small portion (2.64%) of the subtopics located in Geometry area. As this paper deals with the theme 'angles', we will be carrying out the search in the context of the nine texts pointed out at table 03. In this way, we will categorize, according to Table 04, the theme 'Angles'.

Table 04: Classification by Geometry area (Angles).

Paper title and author.	n.	Teaching Segment	Theoretical or Teaching area?	Problematicization? Intramathematical or Contextualized?	Main subthemes.
Marking an angle without a protractor ¹² (Alexandre Kleis)	11	Middle School	Teaching area	Yes Intramathematical	$2\pi R \leftrightarrow 360^\circ$ (arc/angle)
The clock problem ¹³ (Antonio L. P. Pastor)	11	Middle School	Teaching area	Yes Intramathematical	Angles in the Circumference. Sexagesimal system. Direct proportionality.
Still the hands of the clock ¹⁴ (Carlos G. T. A. Moreira)	15	Middle School	Theoretical	Yes Intramathematical	Angles in the Circumference. Sexagesimal system. Direct proportionality.
The concept of the angle ¹⁵ (Cláudio Arconcher)	37	Middle School	Theoretical	No Intramathematical	Angles in the Circumference.
Transformations in the plane and articulated systems ¹⁶ (Sergio D. Assumpção <i>et al.</i>)	47	Middle School	Theoretical	Yes Intramathematical	Ampliation and rotation (R^2)
The Z rule ¹⁷ (André Luis dos Santos)	56	Middle School	Teaching area	No Intramathematical	Parallelism between straight lines;Thales' theorem.
Angles between hands on a clock ¹⁸ (Fernando H. A. Araújo)	72	Middle School	Teaching area	Yes Intramathematical	Angles in the Circumference. Sexagesimal system. Direct proportionality.
Mathematics and origami (Hideo Kumayama)	75	Middle School	Theoretical	Yes Intramathematical	Trigonometry in the Right Triangle.
Penalty!!! (Sérgio Dias Assumpção)	77	Middle School	Theoretical	Yes Contextualization	Parallelism between straight lines; Sum of angles in a triangle; Thales' theorem; Pythagorean theorem.

Source: The author.

Considering table 04, we concluded that the nine texts of the Mathematics Teacher's Magazine seemed to be related to the Middle School. We must remember that the magazine's texts do not mention the teaching segment, so we needed to estimate this level according to the message content. Also, table 04 tell us that 55,55% of the texts were theoretical , while 44,45% were related to teaching area, in a almost equalized situation.

In the problematization sense, table 04 indicates that 66,67% of the texts had chosen this perspective, which is very interesting in the teaching area. Also, the main subtheme were 'Angles in the Circumference' and 'Sexagesimal system', 'Direct proportionality', followed by 'Thales' theorem'.

In Table 05, we correlate the subthemes indicated in Table 04, through the relationship with the specific skills described at the National Common Curricular Base (BNCC), according to Brazil (2018), synthesized at Table 01. We observe that this was not done by the Mathematics Teacher's Magazine.

¹² Marcando um ângulo sem transferidor.

¹³ O problema do relógio.

¹⁴ Ainda os ponteiros do relógio.

¹⁵ O conceito do ângulo.

¹⁶ Transformações no plano e sistemas articulados.

¹⁷ A regra do Z.

¹⁸ Ângulos entre ponteiros de um relógio.

Table 05: Relationship between the subthemes and the BNCC's skills.

Item	Quantity	Percentage	Skill-BNCC
Ampliation and rotation.	1	5,56%	(EF05MA18)
Angles in the Circumference.	4	22,22%	(EF09MA11)
Parallelism between straight lines.	2	11,10%	(EF07MA19)
Direct proportionality.	3	16,67%	Various skills
Sexagesimal system.	3	16,67%	No correspondence
Sum of angles in a triangle.	1	5,56%	(EF07MA20)
Phytagorean theorem.	1	5,56%	(EF09MA13)* ¹⁹ and (EF09MA14)* ²⁰
Talesa theorem.	2	11,10%	(EF07MA19)
Trigonometry in the Right Triangle.	1	5,56%	(EF09MA13)*

Source: The author.

In resume, regarding the correlations of the item 'Angles' with other Mathematics topics, we found that the most frequent exploration was the topic Angles in the Circumference, which included four papers, corresponding to 22.22% of the total. Next, there is two themes, Direct proportionality and the Sexagesimal system, with 16.67% each. Continuing, the items Parallelism between lines and Tales' Theorem appear in two papers, with a frequency of 11.11% each. Finally, with 5.56% each, there are the connections of the theme Angles with Ampliation and rotation in R^2 , the Sum of angles in a triangle, the Pythagorean Theorem and Trigonometry in the Triangle Rectangle.

Some items in the table have been highlighted with a blue asterisk (EF09MA13 and EF09MA14). The reason was that the Brazilian curriculum document (the National Common Curricular Base) does not make an explicitly relation to the theme 'Angles'. This is the case regarding the items 'Pythagorean Theorem' and 'Trigonometry in the Right Triangle', which have specific skills within Geometry area, but the document cited (the National Common Curricular Base) does not mention the relationship with the topic 'angles'. We think that the lack of explanation in the aforementioned document might be due to the fact that these relationships are within the domain of the teacher knowledge.

Direct proportionality is one of the fundamental ideas mentioned at the National Common Curricular Base, and that produce connections with the others: equivalence, order, interdependence, representation, variation and approximation. Direct proportionality is not mentioned within the Thematic Unit 'Geometry', but it is part of the expected connections between Geometry, Arithmetic and Algebra.

We found no mention of the item 'Sexagesimal system' at the National Common Curricular Base. Finally, we summarized some investigating questions surrounding the theme 'angles', which are available in Table 06.

¹⁹ (EF09MA13) Demonstrate metric relationships of the right triangle, including Pythagoras' theorem, including the use of similar triangles.

²⁰ (EF09MA14) Solve and develop problems applying Pythagoras' theorem or proportionality relations involving parallel lines cut by secants.

Table 06: Study of the proposed questions.

(a1) Which segment is the text aimed at (Elementary School or Middle School)?	Elementary School			Middle School	
	1			8	
(a2) Which year is the text aimed at?	5	6	7	8	9
	1	1	3	1	3
(b) Is the paper a theoretical mathematical text or does it cover teaching or learning in basic education?	Theoretical			Teaching and Learning	
	5			4	
(c1) Does the text problematize?	Yes			No	
	6			3	
(c2) Does the text carry out contextualization or is situated in a intramathematical presentation?	Contextualization			Intramathematical	
	1			8	

Source: The author.

In summary, regarding to the theme 'Angles', the Mathematics Teacher's Magazine has presented nine papers since its creation in 1982. It is a very small number given the large lapse of time up to 2021. In the specific subject of 'Angles' we observed that, naturally, the vast majority are related to the segment of Middle School.

To determine the academic year that the papers could probably be addressed, we based on the correlation with the ability related to the Brazilian Curricular Document, given at Brazil (2018). In this sense, there was a higher percentage of production in relation to the 7th grade of Middle School and 9th initial year of High School grade.

Concerning the nine papers that were selected, five of them present a theoretical mathematical text, which seeks to provide a more formal presentation of knowledge. Yet, four other papers seek to establish a relationship with teaching or learning in basic schooling.

The problematization item was worked on six papers. However, the papers problematization was situated in an intramathematical level, revealing few connections with contextualization or applicability, which reveals a horizon far from the necessary learning conditions for students.

5 FINAL CONSIDERATIONS

First of all, this research hypothesized that Mathematics magazines represent an item to be investigated. The use of this kind of online material has consonance with Peter's researches, author who argues the importance to make academic studies about reflection and consultation magazines in many areas.

The analysis of the material that makes up the Mathematics Teacher's Magazine revealed that the most developed Geometry contents were triangles, solids, polygons, geometrical constructions and circle. Regarding to texts themes, we realized that the main discourses were around the curriculum, epistemology, didactics and teacher education in the segment of the Middle School.

The quantitative data obtained from the Mathematics Teacher's Magazine showed 2,64%

of the theme angles among the papers considering Geometry. This is a very thin percentage, if we consider the large list of topics linked with the theme 'angles' as described at the National Common Curricular Base, related at Brazil (2018). This Brazilian curricular document told us that these topics must be present in various degrees of the Middle School. So, we concluded that there were very little publications about the theme angles in Mathematics Teacher's Magazine.

Concerning the texts that permeated the subtheme 'Angles', 22.22% dealt with 'Angles in the Circumference', 16.67% placed 'Direct Proportionality', 16.67% the 'Sexagesimal System', 11.10% dealt with 'Parallelism between straights', 11.10% the 'Tales Theorem', 5.56% the 'Sum of Angles in a Triangle', 5.56% the 'Pythagoras Theorem' and 5.56% to 'Trigonometry in Right triangle'.

Our analysis showed that the papers that are connected to the angles theme from Mathematics Teacher's Magazine had a double characterization. There are papers that place an axiomatic concern on the exposition of mathematical knowledge, so not take in account didactic considerations. And, in the other hand, some that seek to present the themes in a way more linked to the pragmatic, to the context and to explanations that could be more accessible to Middle School students.

Of the papers focused on teaching or learning through pragmatic problems, we noticed that less axiomatic language was used, so there is concern in presenting teachers the possibility of application of Angles' theme in the classroom. However, this applicability was more situated in an intradisciplinary level. So, we expect that the notes we made can instigate mathematics teachers to consult and make use of specialized magazines to prepare their classes in the segment of Elementary School.

This study was directed to the Mathematics Teacher's Magazine, a mathematics magazine in order to highlight the relevance of the topic of angles for the continuing education of teachers. We think that this magazine could be used as an alternative material to the continued training of Mathematics teachers in basic education.

In this sense, we consider that could be make future researches on mathematics magazines. We observe that there are other sources to make academic studies over mathematics researches. For instance, we can investigate if and in what way Brazilian teachers or Mathematics bachelor's degree students use this kind of material in classroom. Another research possibility is to make some mapping over mathematics magazines (what subjects they talk about?, or what the way they treat the subjects?), among other possibilities.

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